CONSERVATIVE DISCONTINUOUS GALERKIN METHODS FOR THE GENERALIZED KORTEWEG-DE VRIES EQUATION

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Abstract. We construct, analyze and numerically validate a class of conservative discontinuous Galerkin (DG) schemes for the Generalized Koreteweg-de-Vries (GKdV) equation

$$\begin{cases} u_t + (u^{p+1})_x + \epsilon u_{xxx} &= 0, & 0 < x < 1,\ 0 < t \leq T, \\ u(x,0) &= u^0(x), & 0 < x < 1, \end{cases}$$

with periodic boundary conditions on the interval $[0,1]$, where $p \geq 1$ is an integer and ϵ a positive parameter. The schemes preserve the first two invariants (the integral and L^2 norm) of the numerical approximations. Existence and uniqueness of the approximations as well as quasi optimal rates of convergence for the errors are proved. We provide numerical evidence that this property imparts the approximations with beneficial attributes such as a more faithful reproduction of the amplitude and phase of traveling wave solutions. We also provide evidence of linear growth of the errors as a function of time.