A hyperbolic inverse problem with angular control on the coefficients

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Abstract

Suppose $q_i(x)$, $i = 1, 2$ are smooth functions on $\mathbb{R}^3$ and $U_i(x,t)$ the solutions of the initial value problem

$$\begin{aligned} \partial_t^2 U_i - \Delta U_i - q_i(x) U_i = \delta(x,t), \qquad (x,t) \in \mathbb{R}^3 \times \mathbb{R} \\ U_i(x,t) = 0, \qquad \text{for } t < 0. \end{aligned}$$

Pick R, T so that $0 < R < T$ and let C be the vertical cylinder $\{(x,t) : |x| = R,\ R \leq t \leq T\}$. We show that if $(U_1, U_{1r}) = (U_2, U_{2r})$ on C then $q_1 = q_2$ on the annular region $R \leq |x| \leq (R+T)/2$ provided there is a $\gamma > 0$, independent of r, so that

$$\int_{|x|=r} |\Delta_S (q_1 - q_2)|^2 \, dS_x \leq \gamma \int_{|x|=r} |q_1 - q_2|^2 \, dS_x, \qquad \forall r \in [R, (R+T)/2].$$

Here Δ_S is the spherical Laplacian on $|x| = r$.